# School of Mathematics Groups



Ben Leimkuler



Max Ruffert

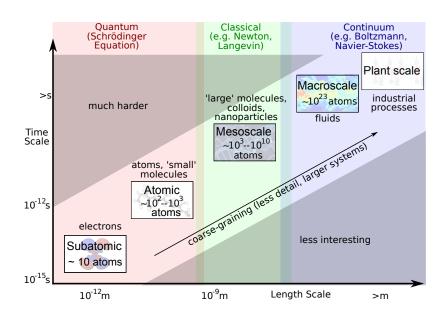


Kostas Zygalakis



Ben Goddard

#### Physical Areas of Interest



• Modelling:

'The development of a mathematical representation of a physical (biological, chemical, economic, data, ...) situation.'

- Quantum
- Classical
- Deterministic
- Stochastic
- ODEs, PDEs, SDEs, Integro-PDEs

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'The development of a mathematical representation of a physical (biological, chemical, economic, data, ...) situation.'

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#### Numerical Analysis:

'The design and analysis of techniques to give approximate but accurate solutions to hard problems.'

- Algorithm design and development
- Rates of convergence
- Propagation of errors
- Numerical stability and well-posedness
- Why your algorithm fails/is spectacularly good in 'special' cases

- Software Development: Some existing projects:
  - ExTASY: Extensible Tools for Advanced Sampling and Analysis Novel MD calculations to provide order-of-magnitude improvements in sampling.
  - MD.M: Molecular Dynamics for MATLAB
     Accessible Langevin/Brownian dynamics simulations, in conjunction with Ben L's book.
  - 2DChebClass
     MATLAB pseudospectral scheme for the solution of stiff, non-local, non-linear PDEs.
  - Superadiabatic quantum molecular dynamics
     MATLAB solution of dynamics through avoided crossings.
  - Implementation of novel algorithms in standard packages such as NAMD, LAMMPS, Gromacs

- Interdisciplinary Research
   Currently have collaborations with (at least):
  - Physicists
  - Chemists
  - Biologists
  - Engineers
  - Informaticians
  - Data scientists

Keen to have more!

#### **MIGSAA**

The Maxwell Institute Graduate School in Analysis and its Applications.

- 12 students per year
- 2 more years of intake
- Currently have 2 MIGSAA students working in MD-related areas (and many more in Applied Maths generally)
- Theme is very broadly-interpreted
- Very happy to have interdisciplinary co-supervision
- Good opportunity to kick-start a collaboration



Variety of work on formulation and stochastic numerical methods for MD simulation in classical biomolecular and mesoscale regimes, including thermostats, barostats, dissipative particle dynamics, Brownian/Langevin dynamics

Current/Recent projects

Langevin numerical method

Constant temperature and pressure MD

Adaptive thermostats for nonequilibrium MD

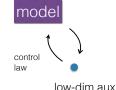
Geodesic integrator for constrained dynamics

Continuous tempering (multiple temperature) strategies

Ensemble quasi-Newton/preconditioned dynamics

Isokinetic stochastic methods for multiple timestepping

(100fs stepsizes for MD!)



dynamics

# Adaptive Alternatives to DPD

PAdL L. & Shang, J. Comput. Phys., 2016

 $d\mathbf{q} = \mathbf{M}^{-1}\mathbf{p}dt$ 

 $d\xi = G(\mathbf{q}, \mathbf{p})dt$ 

 $d\mathbf{p} = \mathbf{F}(\mathbf{q}, t)dt - \xi \mathbf{\Gamma}(\mathbf{q})\mathbf{M}^{-1}\mathbf{p}dt + \sigma \mathbf{\Sigma}(\mathbf{q})d\mathbf{W}$ "Adaptive variant of DPD"

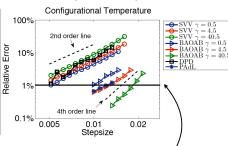
Submitted: Assessing numerical methods for molecular and particle simulation

w. X. Shang (Brown) and M. Kroeger (ETH)

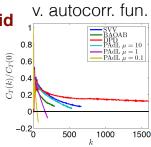
PAdL compared to other schemes for sheared polymer melts

PAdL is typically very accurate for configuration dependent quantities (functions of position)

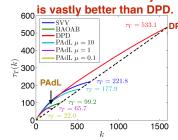
# FENE type Polymer Melts



Much more rapid convergence to equilibrium



PAdL simulation cost to ach. fixed accuracy is vastly better than DPD



# Zygalakis Group



Kostas Zygalakis

### Numerical Analysis of SDEs

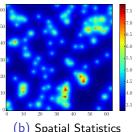
$$dX_t = \nabla \log \pi(X_t) + \sqrt{2}dW_t$$

- Specific type of behaviour as  $t \to \infty$  (ergodicity)
- How to discretise such equations? Design and analysis of numerical methods capable of capturing the long time behaviour

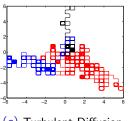
# **Applications**



(a) Machine Learning

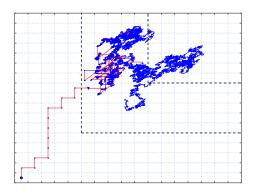


(b) Spatial Statistics



(c) Turbulent Diffusion

# Hybrid modelling of stochastic chemical kinetics



# **Goddard Group**



Ben Goddard



Miguel Duran Olivencia



Tim Hurst



Rory Mills

### Goddard Group

Modelling, analysis and numerics for a wide range of systems including electronic structure theory, quantum MD and soft matter/complex fluids.

#### Current projects:

- Statistical mechanics for:
  - soft matter
  - granular media
  - biological systems
- Free energy approximations
- Langevin dynamics, especially with hydrodynamic interactions
- Pseudospectral methods numerical schemes
- Quantum molecular dynamics with avoided crossings

# Goddard Group: Dynamical Density Functional Theory

Statistical mechanics for complex fluids.

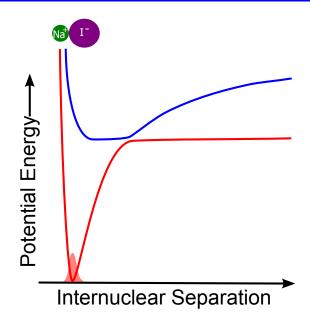
Especially interested in hydrodynamic interactions.

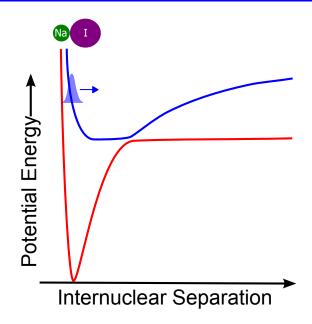
$$\partial_{t}\rho(\mathbf{r},t) = \frac{D_{0}}{k_{B}T} \left[ \rho(\mathbf{r},t)\mathbf{W}_{0}(\mathbf{r},t)\mathbf{\nabla}_{\mathbf{r}} \frac{\delta\mathcal{F}[\rho]}{\delta\rho(\mathbf{r},t)} + \int d\mathbf{r}'\rho(\mathbf{r},t)\rho(\mathbf{r}',t)g([\rho];\mathbf{r},\mathbf{r}')\mathbf{W}_{11}(\mathbf{r},\mathbf{r}')\mathbf{\nabla}_{\mathbf{r}} \frac{\delta\mathcal{F}[\rho]}{\delta\rho(\mathbf{r},t)} + \int d\mathbf{r}'\rho(\mathbf{r},t)\rho(\mathbf{r}',t)g([\rho];\mathbf{r},\mathbf{r}')\mathbf{W}_{12}(\mathbf{r},\mathbf{r}')\mathbf{\nabla}_{\mathbf{r}'} \frac{\delta\mathcal{F}[\rho]}{\delta\rho(\mathbf{r}',t)} \right]$$

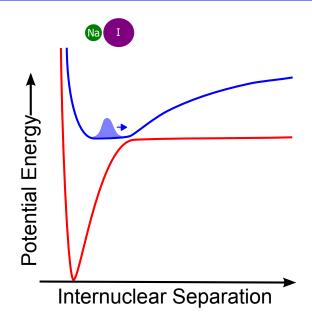
With S. Kalliadasis (Imperial), Y. Kevrekidis (Princeton), G. Pavliotis (Imperial), D. Sibley (Loughborough), . . .

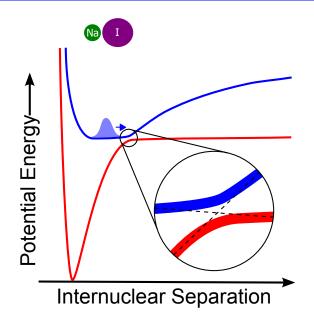
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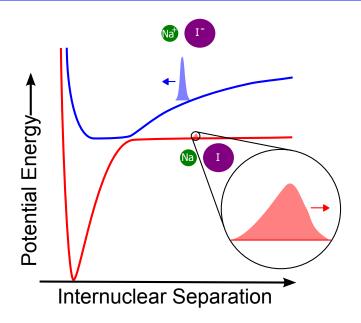
# Goddard Group: Dynamical Density Functional Theory

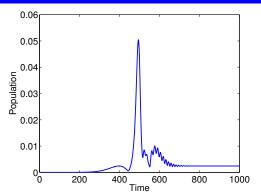








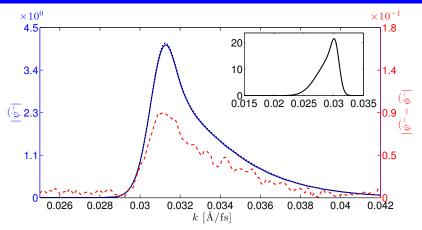




$$\widehat{\psi_n^-}^{\varepsilon}(k,t) \approx e^{-\frac{i}{\varepsilon}t\hat{H}^-} \frac{\eta^* + k}{2|\eta^*|} e^{-\frac{\tau_c}{2\delta\epsilon}|k-\eta^*|} e^{-i\frac{\tau_r}{2\delta\epsilon}(k-\eta^*)} \widehat{\phi}^{\varepsilon}(\eta^*) \chi_{k^2 > 4\delta}$$

Only requires 1-level dynamics and adiabatic PES.

With V. Betz (Darmstadt), U. Manthe (Bielefeld), S. Teufel (Tübingen).



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